# Judgment Aggregation and the Discursive Dilemma

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Judgment aggregation theory investigates which procedures a group could or should use to form collective judgments ('yes' or 'no') on a given set of propositions or issues, based on the judgments of the group members. How, for instance, should the citizens of a state reach collective judgments on the three propositions that multiculturalism is desirable, that immigration should be promoted, and that the former implies the latter? And how should a jury in court form collective judgments on the propositions that the defendant has broken the contract, that this contract was legally valid, and that the defendant is liable to pay damages?

# The discursive dilemma and political philosophy

Such collective decision problems are vulnerable to the *discursive dilemma*, a phenomenon generalizing the *doctrinal paradox* in jurisprudence. The source of the dilemma is that the propositions under consideration are logically interconnected. In our first example, the third proposition is a conditional involving the first two propositions; and in our second example, the third proposition is equivalent to the conjunction of the first two propositions (according to the generally acknowledged legal doctrine that breach of a valid contract is necessary and sufficient for liability). The initially most natural and democratically appealing procedure – proposition-wise majority voting – may generate inconsistent collective judgments. In the case of our first example, Table 1 illustrates a situation in which the population is split into three camps such that, overall, a majority believes that multiculturalism is desirable (proposition *P*), another majority believes that *if* multiculturalism is desirable *then* immigration should be promoted (proposition *if-P-then-Q*), but yet another majority believes that immigration should *not* be promoted.

	P	If-P-then-Q	Q
1/3 of the population	Yes	Yes	Yes
1/3 of the population	Yes	No	No
1/3 of the population	No	Yes	No
The majority	Yes	Yes	No

Table 1: Inconsistent majority judgments

This phenomenon of majoritarian inconsistencies poses a serious challenge to the very meaning and possibility of democracy, since it seems that collective judgments cannot be both consistent and democratically responsive to people's judgments.

To restore collective rationality, two routes are often contrasted. Under the *premise-based* route, the collective adopts the majority-supported judgments P and if-P-then-Q (interpreted as two *premises*), from which it derives the judgment Q (interpreted as a *conclusion*). Under the *conclusion-based* route, the collective instead adopts the majority-supported judgment not-Q, and either forms no judgments at all on the premise propositions, or forms some judgments on them which are logically consistent with not-Q, such as the judgments not-P and if-P-then-Q. In short, the premise-based approach respects majorities on premises while overruling majorities on conclusions, while the conclusion-based approach does the converse.

In response to the discursive dilemma, a highly interdisciplinary body of research has developed, conducted mainly by economists, philosophers, political scientists and computer scientists. The less formal branch of research has its home in political philosophy. It focuses on the nature and role of the collective agent and the extent to which such an agent should provide reasons (premises) for its policies (conclusions). As is sometimes argued from the perspective of republican democratic theory, the state must act upon and publicly provide reasons in order for its actions to be contestable. Contestability of state actions is in turn important for preventing arbitrary state interference in citizen's lives, i.e., to render citizens free in the republican sense. The need for reason-based state actions or policies is often taken to imply the superiority of premise-based over conclusion-based aggregation.

# The formal theory of judgment aggregation

The more formal area of research stands in the tradition of Arrovian social choice theory. The judgment aggregation problem is formulated in full abstract generality. Two central ingredients of the theory are, firstly, the group's *agenda*, i.e., the set of propositions on which judgments are formed; and secondly, the notion of an *aggregation rule* or *procedure*, i.e., a function which takes each person's set of judgments as input and returns a collective set of judgments. Simple examples of aggregation rules are proposition-wise majority rule, proposition-wise quota rules (with acceptance thresholds that may differ from the majority threshold and may vary across propositions), premise-based voting, conclusion-based voting, and the 'expert rule' which universally adopts the judgments of a fixed individual (the 'expert' or 'dictator'). The generality of the framework stems from the fact that virtually any kind of decision can be construed as the formation of judgments on particular propositions. Notably, the classical preference aggregation problem in social choice theory emerges a special case,

because a preference relation can be construed as a set of judgments on propositions of the form 'x is better than y', where x and y denote choice alternatives.

# The axiomatic approach

Within judgment aggregation theory, one may broadly distinguish between an axiomatic and a constructive approach, the two of which go hand in hand. The axiomatic approach starts by formulating general requirements ('axioms') on aggregation rules which capture normative principles or intuitions. An example of an axiom of procedural fairness is anonymity, which forbids differential treatment of voters, i.e., requires that the collective judgment set only depends on the number of individuals holding each given judgment set, regardless of their identity. This axiom for instance excludes the mentioned expert rule. The axiom of consistency of collective judgment sets excludes proposition-wise majority rule, as the discursive dilemma shows. Once a set of axioms is specified, one proceeds by determining all judgment aggregation rules satisfying the axioms, a more or less difficult mathematical exercise. Ideally, there is a single such rule, but often there are many rules (leaving a choice to be made) or no rules (forcing one to abandon an axiom). Indeed, in a series of *impossibility theorems*, it has been established that various combinations of axioms are not satisfied by any aggregation rule if the agenda of propositions is sufficiently complex. Many of these theorems are in a similar spirit as Kenneth Arrow's famous impossibility theorem in preference aggregation theory. Indeed, one of them stands out as being an exact generalization of Arrow's theorem from preference aggregation problems to arbitrary judgment aggregation problems. A quite different impossibility theorem generalizes Amarya Sen's influential 'Impossibility of a Paretian Liberal'; it brings to light a conflict between respecting unanimous judgments and respecting the right of individuals or minorities to alone determine the collective judgment on propositions within the private sphere or the area of special competence.

### The constructive approach

This approach tries directly to devise concrete aggregation rules for reaching consistent and democratically responsive collective judgments, without a preceding axiomatic derivation. The following salient proposals or paradigms can be contrasted: (i) *premise*-or *conclusion-based* aggregation rules; (ii) *quota rules* with well-calibrated acceptance thresholds; (iii) *sequential* rules, where the propositions in the agenda are voted upon one by one in an order of priority and where the vote on any proposition is suspended if the previous voting outcomes on propositions of higher priority already imply a judgment ('yes' or 'no') on the current propositions; (iv) *distance-based* rules, where the collective adopts a consistent set of judgments whose sum-total distance to people's sets of judgments is as small as possible, with respect to some distance measure between judgment sets; (v) *scoring* rules, where the collective adopts a consistent set of

judgments which receives maximal sum-total score from the individuals, with respect to some definition of 'scores'.

# Localistic versus holistic aggregation

Under a *localistic* (or *proposition-wise*) understanding of democratic responsiveness, the collective judgment on any given proposition should be formed solely on the basis of people's judgments on this proposition, independently of their judgments on other propositions. By contrast, under a *holistic* conception of democratic responsiveness, the collective judgment on, say, whether immigration should be promoted may be influenced by people's judgments on other propositions, such as 'premises' or even unrelated propositions about taxation. Here, even an overwhelming majority judgment on whether immigration should be promoted may be overruled in the name of people's judgments elsewhere.

Localism is the content of the (controversial) *independence* axiom, the counterpart in judgment aggregation theory of the (equally controversial) axiom of 'independence of irrelevant alternatives' in preference aggregation theory. A virtue of independence is that it is necessary for preventing the manipulation of outcomes through certain types of strategic voting or strategic agenda setting, as was proved. However, independence features as the central axiom in most impossibility theorems. Hence, the goal of aggregating localistically is unachievable (for agendas subject to the impossibility result) – whether or not localism is normatively desirable.

#### The procedural versus the epistemic approach

Two contrasting approaches or aims may be pursued when designing the aggregation rule. The *procedural* approach aims for a 'procedurally fair' rule; for instance, anonymity is usually a central procedural virtue. The *epistemic* approach aims for a rule which generates 'correct' or 'true' collective judgments by an external, procedure-independent standard of correctness or truth; here, anonymity may be violated in the name of differences in information or competence. While most of the literature has a proceduralist flavour, some work takes the epistemic perspective and stands in the tradition of Condorcet's Jury Theorem.

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# **Cross-references to other Encyclopedia articles:**

Collective Agents
Collective Rationality
Decision Theory
Group Beliefs
Sen's Paretian Liberal
Social Choice Theory
Social Epistemology

## **Further readings:**

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